

Analysis of Variance Table

in

Simple Linear Regression

Applied Regression and Other Multivariable Methods
Sections 7-1 - 7-2

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ANOVA Table

- Overall summary table for any regression analysis
- Table consists of several variance estimates
- Can use these estimates to assess
 - Is the true slope zero?
 - What is the strength of the straight line model?
 - Is a straight line model appropriate?

Constructed by decomposing the total sum of squares about the mean (i.e., correct for the mean) into the sum of squares explained by the regression model and the unexplained residual sum of squares (i.e., what's left over)

$$\begin{aligned} \sum (Y_i - \bar{Y})^2 &= \sum (Y_i - \hat{Y}_i + \hat{Y}_i - \bar{Y})^2 \\ &= \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)^2 \\ SSY &= (SSY - SSE) + SSE \end{aligned}$$

7-1

Degrees of Freedom & Mean Squares

- For each SS, there is associated df
- For simple linear regression
 - $df_{Total} = n - 1$
 - $df_{Regression} = 1$
 - $df_{Error} = n - 2$
- A mean square is defined to be SS/df
- Mean squares provide variance estimates
 - $SSY/(n - 1) = S_{\bar{Y}}^2$
 - $SSE/(n - 2) = S_{Y|X}^2$
 - $(SSY - SSE)/1$ estimates $\sigma_{Y|X}^2$ under $H_0: \beta_1 = 0$
- Use ratio of mean squares to perform tests
- $F = (SSY - SSE)/MSE \sim F_{1, n-2}$ under H_0

7-2

F-test in Simple Linear Regression

- $F = (SSY - SSE)/MSE$ same as t-test for β_1
- Can show
 - $T^2 = (\hat{\beta}_1 / S_{\hat{\beta}_1})^2 = F$
 - $F_{1, v} = t_v^2$
- In example below $21.33 = 4.62^2$

Dependent Variable: sbp

Source	DF	Analysis of Variance		F Value	Pr > F
		Sum of Squares	Mean Square		
Model	1	6394.02269	6394.02269	21.33	<.0001
Error	28	8393.44398	299.76586		
Corrected Total	29	14787			
Root MSE		17.31375	R-Square	0.4324	
Dependent Mean		142.53333	Adj R-Sq	0.4121	
Coeff Var		12.14716			

Variable	DF	Parameter Estimates		t Value	Pr > t
		Parameter Estimate	Standard Error		
Intercept	1	98.71472	10.00047	9.87	<.0001
age	1	0.97087	0.21022	4.62	<.0001

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