

Blocking and Confounding in 2^k Factorial

Design of Experiments - Montgomery
Chapter 7

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Blocking in 2^k Factorial Designs

- For RCBD, each combination run in each block
 - $2^2 \rightarrow 4$ EUs per block
 - $2^3 \rightarrow 8$ EUs per block
 - Randomize run order within block
- Suppose you cannot run all comb. within block
- Must do some sort of incomplete block analysis
- If you do not, certain effects confounded
- Confounding: two effects are indistinguishable
- May "sacrifice" certain effects thought to be small
- 2^k design makes set-up simple

24-1

Confounding in 2^k with only 2 blocks

- Blocks assumed to allow 2^{k-1} combinations
- First consider 2^2 factorial (2 combs per blk)
- Possible pairings
 - 1 (1) and b together \rightarrow a and ab together
 - 2 (1) and a together \rightarrow b and ab together
 - 3 (1) and ab together \rightarrow a and b together
- 1 Effect of A, $(ab+a-b-(1))/2$, is block difference
- 2 Effect of B, $(ab-a+b-(1))/2$, is block difference

Both have a main effect confounded with block
- 3 Effect of AB, $(ab-a-b+1)/2$, is block difference

Allows for main effect estimates (blks cancel out)

24-2

Choice of Confounding Factors

- Common to confound highest order interaction
- Can use +/- table to determine blks
- For two factor, recall the following table

A	B	AB	Symbol
-	-	+	(1)
+	-	-	a
-	+	-	b
+	+	+	ab

- Use confounding column to determine blks

+ 's in blk 1 and - 's in blk 2

- Consider three factor

A	B	C	AB	AC	BC	ABC	Symbol
-	-	-	+	+	+	-	(1)
+	-	-	-	+	+	+	a
-	+	-	-	+	-	+	b
+	+	-	+	-	-	+	ab
-	-	+	+	-	-	+	c
+	-	+	-	+	-	-	ac
-	+	+	-	-	+	-	bc
+	+	+	+	+	+	+	abc

- Best assignment would be a,b,c,abc together
- Can estimate all but three factor interaction

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2^k Factorial in Four Blocks

- Four blocks each containing 2^{k-2} EUs
- Useful in situations where $k \geq 4$
- Must select two effects to confound
- Will result in a third confounded factor
- Consider 6 factor factorial run in 4 blocks of 16 EUs
 - Block 1 uses ABC + and DEF +
 - Block 2 uses ABC + and DEF -
 - Block 3 uses ABC - and DEF +
 - Block 4 uses ABC - and DEF -
- Results in (ABC)(DEF)=ABCDEF confounded
 - ABCD and DEF → ABCEf confounded
 - AB and ABEF → EF confounded
- Can extend to 8 and 16 blks
- Table 7-8 summarizes these designs (pg 298)

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Partial Confounding

- Can replicate blocking design
- Confound different effects each replication
- Allows estimation of all effects
 - Confounded effects based on nonconfounded replicates
 - Can use Yates' Algorithm for all nonconfounded effects
 - See Example 7-3 (pg 300)

```

\* Example 7-3 *\
data cool;
input block fact1 fact2 fact3 y;
cards;
  1 -1 -1 -1 -3
  1  1  1 -1  2
  ...
;

proc glm;
class fact1 fact2 fact3 block;
model y= block fact1|fact2|fact3;
    
```

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Class	Levels	Values
FACT1	2	1 -1
FACT2	2	1 -1
FACT3	2	1 -1
BLOCK	4	1 2 3 4

Dependent Variable: Y

Source	DF	Sum of Squares	F Value	Pr > F
Model	10	74.25000000	9.90	0.0103
Error	5	3.75000000		
Corrected Total	15	78.00000000		

Source	DF	Type I SS	F Value	Pr > F
BLOCK	3	3.50000000	1.56	0.3101
FACT1	1	36.00000000	48.00	0.0010
FACT2	1	20.25000000	27.00	0.0035
FACT1*FACT2	1	0.50000000	0.67	0.4513
FACT3	1	12.25000000	16.33	0.0099
FACT1*FACT3	1	0.25000000	0.33	0.5887
FACT2*FACT3	1	1.00000000	1.33	0.3004
FACT1*FACT2*FACT3	1	0.50000000	0.67	0.4513

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Example

Consider a 2³ factorial run in 4 blocks

Each replicate will result in 3 confounded effects

Consider 4 replicates for 32 total observations

Replicate 1: Confound BC and AC → AB

Replicate 2: Confound BC and ABC → A

Replicate 3: Confound AC and ABC → B

Replicate 4: Confound AB and ABC → C

Three replicates to estimate A, B, and C

Two replicates to estimate AB, AC, and BC

One replicate to estimate ABC

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Data

Replicate 1 - AB, AC, BC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
75 (1)	89 ab	61 a	30 b
100 abc	73 c	45 bc	54 ac

Replicate 2 - A, BC, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
60 (1)	47 a	1 b	26 ac
34 bc	81 abc	35 c	52 ab

Replicate 3 - B, AC, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
58 (1)	48 a	18 b	68 ab
42 ac	52 c	82 abc	32 bc

Replicate 4 - C, AB, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
47 (1)	34 a	50 c	37 ac
57 ab	4 b	80 abc	27 bc

Since no effect is estimated from all replications (except error), we will compute the effect sums associated with each replicate and combine the appropriate information.

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The following are the column 3 sums using Yates' Algorithm for each of the replicates.

Only the sums that are not confounded with the particular rep are presented

Effect	Rep 1	Rep 2	Rep 3	Rep 4
(1)	527	336	400	336
A	81	-	80	80
B	1	0	-	0
AB	-	120	120	-
C	17	16	16	-
AC	-	0	-	0
BC	-	-	40	40
ABC	1	-	-	-

$$SS_A = \frac{(81 + 80 + 80)^2}{3(8)}$$

$$SS_B = \frac{(1 + 0 + 0)^2}{3(8)}$$

$$SS_{AB} = \frac{(120 + 120)^2}{2(8)}$$

$$SS_C = \frac{(16 + 16 + 17)^2}{3(8)}$$

$$SS_{AC} = \frac{(0 + 0)^2}{2(8)}$$

$$SS_{BC} = \frac{(40 + 40)^2}{2(8)}$$

$$SS_{ABC} = \frac{1^2}{1(8)}$$

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Using SAS

```
options nocenter ps=50 ls=80;

data new;
input repl blk a b c resp;
cards;
1 1 0 0 0 75
1 1 1 1 1 100
1 2 1 1 0 89
1 2 0 0 1 73
...
4 3 0 0 1 50
4 3 1 1 1 80
4 4 1 0 1 37
4 4 0 1 1 27
;

proc glm;
class repl blk a b c;
model resp = repl blk(repl) a|b|c;
```

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SAS Output

Dependent Variable: RESP				
Source	DF	Sum of Squares	Mean Square	F Value
Model	22	17128.718750	778.578125	28028.81
Error	9	0.250000	0.027778	
Corrected Total	31	17128.968750		

Source	DF	Type I SS	Mean Square	F Value
REPL	3	3040.0937500	1013.3645833	36481.13
BLK(REPL)	12	7568.3750000	630.6979167	22705.13
A	1	2420.0416667	2420.0416667	87121.50
B	1	0.0416667	0.0416667	1.50
A*B	1	3600.0000000	3600.0000000	99999.99
C	1	100.0416667	100.0416667	3601.50
A*C	1	0.0000000	0.0000000	0.00
B*C	1	400.0000000	400.0000000	14400.00
A*B*C	1	0.1250000	0.1250000	4.50

Source	DF	Type III SS	Mean Square	F Value
REPL	3	3040.0937500	1013.3645833	36481.13
BLK(REPL)	12	653.9062500	54.4921875	1961.72
A	1	2420.0416667	2420.0416667	87121.50
B	1	0.0416667	0.0416667	1.50
A*B	1	3600.0000000	3600.0000000	99999.99
C	1	100.0416667	100.0416667	3601.50
A*C	1	0.0000000	0.0000000	0.00
B*C	1	400.0000000	400.0000000	14400.00
A*B*C	1	0.1250000	0.1250000	4.50

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