

## Blocking Designs

Design of Experiments - Montgomery  
Section 4-1

11

## Randomized Complete Block Design

- Nuisance factor
  - Has effect on response but effect not of interest
  - If unknown → Protection through randomization
  - If known but uncontrollable → Analysis of Covariance
  - If known and controllable → Blocking Design
- Randomized Complete Block Design (RCBD)
  - $b$  blocks each consisting of  $a$  experimental units
  - $a$  treatments are randomly assigned to EUs within block
  - results in restriction on randomization
  - extension of paired t-test where pairs=blocks

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## Statistical Model

- $b$  blocks and  $a$  treatments
- Statistical model is

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \end{cases}$$

$\mu$  - grand mean

$\tau_i$  -  $i$ th treatment effect

$\beta_j$  -  $j$ th block effect

$\epsilon_{ij} \sim N(0, \sigma^2)$

- Assume additional additive effect due to block  
Block effect the same for all EUs in block
- Treatment and block effects deviations from mean  
Common parameter restrictions are then

$$\sum \tau_i = 0 \quad \sum \beta_j = 0$$

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## Partitioning the SS

- Rewrite observation as:
 
$$y_{ij} = \bar{y}_{..} + (\bar{y}_{i.} - \bar{y}_{..}) + (\bar{y}_{.j} - \bar{y}_{..}) + (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})$$

$$= \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + \hat{\epsilon}_{ij}$$
- Can partition  $SS_T = \sum \sum (y_{ij} - \bar{y}_{..})^2$  into
 
$$b \sum (\bar{y}_{i.} - \bar{y}_{..})^2 + a \sum (\bar{y}_{.j} - \bar{y}_{..})^2 + \sum \sum (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2$$

$$SS_{\text{Treatment}} + SS_{\text{Block}} + SS_E$$
- Under  $H_0$ , all  $SS/\sigma^2$  independent  $\chi^2$
- Ratio of SS will be F distributed

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## Hypothesis Testing

- Can show (in the fixed case):

$$E(MS_E) = \sigma^2$$

$$E(MS_{\text{Treatment}}) = \sigma^2 + b \sum \tau_i^2 / (a - 1)$$

$$E(MS_{\text{Block}}) = \sigma^2 + a \sum \beta_j^2 / (b - 1)$$

- Use F-test to test equality of treatment effects

$$F_0 = \frac{SS_{\text{Treatment}} / (a - 1)}{SS_E / ((a - 1)(b - 1))}$$

- Caution testing block effects

- Usually not of interest (blocked for a reason)
- Blocks not randomized to experimental units (restriction)
- Differing opinions on using F-test
- Can view ratio to see if blocking successful

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## Analysis of Variance Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	$F_0$
Blocks	$SS_{\text{Block}}$	$b - 1$	$MS_{\text{Block}}$	
Treatment	$SS_{\text{Treatment}}$	$a - 1$	$MS_{\text{Treatment}}$	$F_0$
Error	$SS_E$	$(b - 1)(a - 1)$	$MS_E$	
Total	$SS_T$	$ba - 1$		

$$SS_T = \sum \sum y_{ij}^2 - y_{..}^2 / N$$

$$SS_{\text{Treatment}} = \frac{1}{b} \sum y_{i.}^2 - y_{..}^2 / N$$

$$SS_{\text{Block}} = \frac{1}{a} \sum y_{.j}^2 - y_{..}^2 / N$$

$$SS_E = SS_T - SS_{\text{Treatment}} - SS_{\text{Block}}$$

If  $F_0 > F_{\alpha, a-1, (b-1)(a-1)}$  then reject  $H_0$

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## Example

An experiment was designed to study the performance of four different detergents for cleaning clothes. The following "cleanness" readings (higher=cleaner) were obtained with specially designed equipment for three different types of common stains. Is there a difference among the detergents?

	Stain 1	Stain 2	Stain 3
Detergent 1	45	43	51
Detergent 2	47	46	52
Detergent 3	48	50	55
Detergent 4	42	37	49

$$\sum \sum y_{ij} = 565 \text{ and } \sum \sum y_{ij}^2 = 26867$$

$$y_{1.} = 139, y_{2.} = 145, y_{3.} = 153 \text{ and } y_{4.} = 128$$

$$y_{.1} = 182, y_{.2} = 176, \text{ and } y_{.3} = 207$$

$$SS_T = 26867 - 565^2 / 12 = 265$$

$$SS_{\text{Treatment}} = (139^2 + 145^2 + 153^2 + 128^2) / 3 - 565^2 / 12 = 111$$

$$SS_{\text{Block}} = (182^2 + 176^2 + 207^2) / 4 - 565^2 / 12 = 135$$

$$SS_E = 265 - 111 - 135 = 19$$

$$F_0 = (111/3) / (19/6) = 11.6$$

P-value < 0.01 (Reject  $H_0$  - There is a detergent difference)

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## Diagnostics

### Assumptions

- Model is correct (additivity assumption)
- Errors independent, Normally distributed
- Constant variance

### Normality

- Histogram, normal probability plot of residuals

### Variance and Unusual Observations

- Residuals vs blocks
- Residuals vs treatments
- Residuals vs  $\hat{y}_{ij}$

### Additivity

- Residuals vs  $\hat{y}_{ij}$
- If curvilinear, interaction likely exists
- Interaction: Block effect different for different treatments
- Tukey's Test of Non-additivity (Formal Test)
- Usually transform to eliminate interaction

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# Comparisons of Treatments

- Multiple Comparisons/Contrasts
  - Similar procedures as CRD
  - $n$  is replaced by  $b$  in all formulas
  - Degrees of freedom error is  $(b - 1)(a - 1)$
- Example : Comparison of Detergents
  - Duncan's Multiple Range ( $\alpha = .05$ )
  - 6 degrees of freedom error  $\rightarrow r_\alpha = (3.46, 3.58, 3.64)$
  - $s_{\bar{y}} = \sqrt{MSE/3} = \sqrt{(19/6)/3} = 1.03$
  - Least Significant Ranges are (3.56, 3.68, 3.74)

Comparison of Means

	Treatments			
	4	1	2	3
	42.67	46.33	48.33	51.00
	A		C	C
		B	B	

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# Using SAS

```
options nocenter ls=78;
options colors=(none);
symbol1 v=circle; axis1 offset=(5);

data wash;
input stain soap y @@;
cards;
1 1 45 1 2 47 1 3 48 1 4 42
2 1 43 2 2 46 2 3 50 2 4 37
3 1 51 3 2 52 3 3 55 3 4 49
;

proc glm;
class stain soap;
model y = soap stain;
means soap / duncan lines;
output out=diag r=res p=pred;

proc univariate noprint;
qqplot res / normal (L=1 mu=0 sigma=est);
hist res /normal (L=1 mu=0 sigma=est) kernel(L=2 K=quadratic);
run;

proc gplot;
plot res*soap / haxis=axis1;
plot res*stain / haxis=axis1;
plot res*pred;
run;
```

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Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	246.0833333	49.2166667	15.68	0.0022
Error	6	18.8333333	3.1388889		
Corrected Total	11	264.9166667			

R-Square	Coeff Var	Root MSE	y Mean
0.928908	3.762883	1.771691	47.08333

Source	DF	Type I SS	Mean Square	F Value	Pr > F
soap	3	110.9166667	36.9722222	11.78	0.0063
stain	2	135.1666667	67.5833333	21.53	0.0018

Source	DF	Type III SS	Mean Square	F Value	Pr > F
soap	3	110.9166667	36.9722222	11.78	0.0063
stain	2	135.1666667	67.5833333	21.53	0.0018

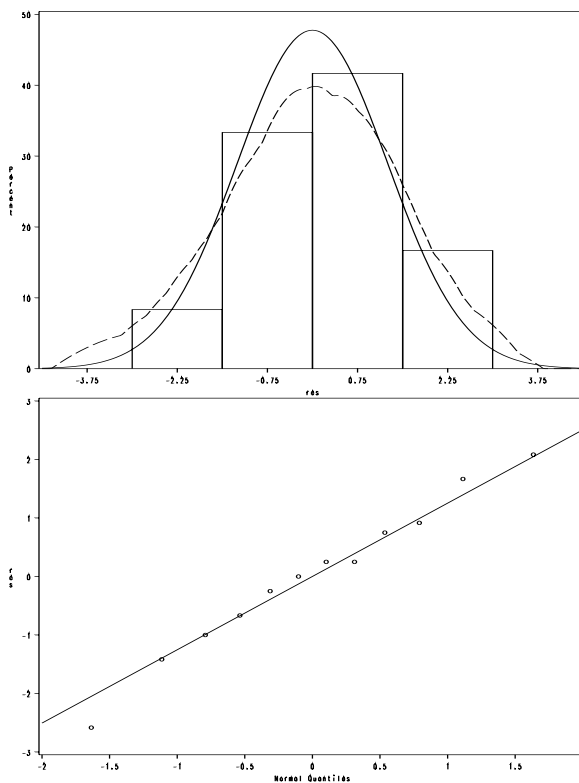
Duncan's Multiple Range Test for y

Alpha	0.05
Error Degrees of Freedom	6
Error Mean Square	3.138889

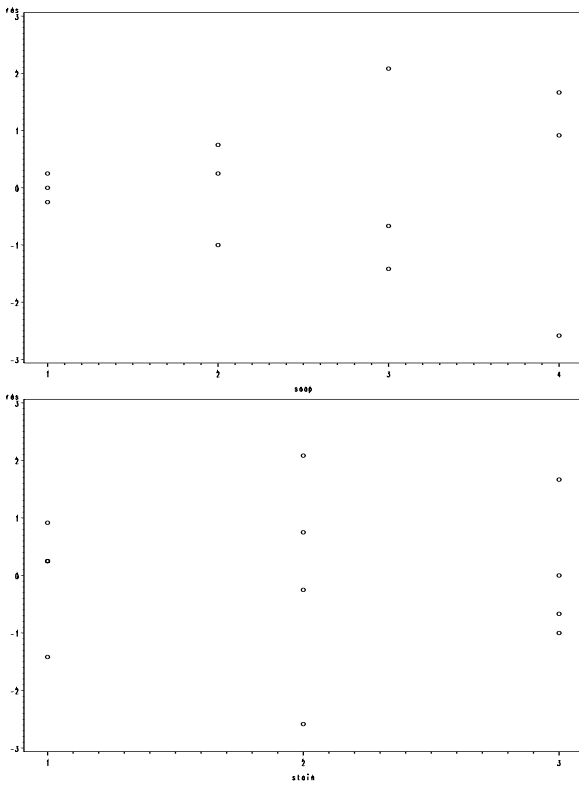
Number of Means	2	3	4
Critical Range	3.540	3.669	3.732

	Mean	N	soap
A	51.000	3	3
A			
B	48.333	3	2
B			
B	46.333	3	1
C	42.667	3	4

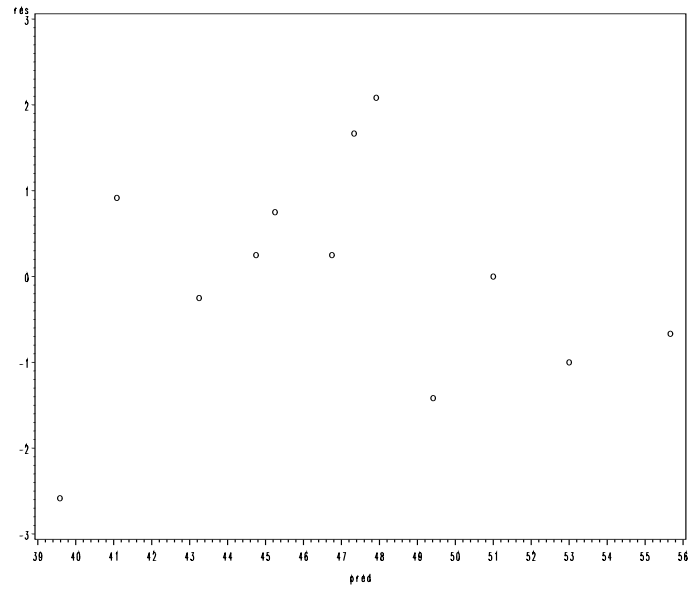
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## Regression Model

- Regression model simple extension of CRD
- Add additional  $b - 1$  columns to represent block
- Block columns orthogonal to treatment columns
- Thus, order of fit does not matter

$$X = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & -1 & -1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & -1 & 0 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

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## Missing Values

- When missing
  - Orthogonality lost - missing row in  $X$
  - Design unbalanced  $\rightarrow$  Order of fit important
- Procedures
  - 1 Exact (Regression) approach
    - Use Type III SS's (general regression signif test)
  - 2 Approximate approach : Estimate missing value
    - Choose value to minimize  $SS_E$  (minimum contribution)
    - Take derivative and set equal to zero

$$SS_E = \sum \sum y_{ij}^2 - y_{..}^2/ab - \frac{1}{b} \sum y_{i.}^2 + y_{i.}^2/ab - \frac{1}{a} \sum y_{.j}^2 + y_{.j}^2/ab$$

$$= x^2 - \frac{1}{b}(y'_{i.} + x)^2 - \frac{1}{a}(y'_{.j} + x)^2 + \frac{1}{ab}(y'_{.} + x)^2 + R$$

$$x = \frac{ay'_{i.} + by'_{.j} - y'_{.}}{(a-1)(b-1)}$$

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## Example

- Consider detergent comparison example
- Suppose  $y_{4,2} = 37$  is missing
- Regression approach
  - Same procedure in SAS - look at Type III result

- Estimate Approach

$$y'_{4,} = 91 \quad y'_{,2} = 528 \quad y'_{,2} = 139$$

- Estimate is

$$x = \frac{4(91) + 3(139) - 528}{6} = 42.17$$

- Do analysis but adjust error degrees of freedom

- Regression:  $\hat{\sigma}^2 = 1.097$
- Estimate:  $\hat{\sigma}^2 = 1.097$  (must divide by 5 not 6)

```
options nocenter ps=60 ls=78;
```

```
data wash;
input stain soap y @@;
if y=37 then y=.;
cards;
1 1 45 1 2 47 1 3 48 1 4 42
2 1 43 2 2 46 2 3 50 2 4 37
3 1 51 3 2 52 3 3 55 3 4 49
;
```

```
proc glm;
classes stain soap;
model y = soap stain;
output out=diag r=res p=pred;
means soap / lsd lines;
lsmeans soap / stderr;
```

```
data new1;
set wash;
if y=. then y=42.166666666;
```

```
proc glm;
classes stain soap;
model y = soap stain;
output out=diag r=res p=pred;
means soap / lsd lines;
run;
```

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## Regression - Type III

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	148.5138889	29.7027778	27.07	0.0013
Error	5	5.4861111	1.0972222		
Corrected Total	10	154.0000000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
soap	3	48.1666667	16.0555556	14.63	0.0066
stain	2	100.3472222	50.1736111	45.73	0.0006

Source	DF	Type III SS	Mean Square	F Value	Pr > F
soap	3	58.9305556	19.6435185	17.90	0.0042
stain	2	100.3472222	50.1736111	45.73	0.0006

t Tests (LSD) for y

	Mean	N	soap
A	51.0000	3	3
B	48.3333	3	2
C	46.3333	3	1
D	45.5000	2	4

Least Squares Means

soap	y LSMEAN	Standard Error	Pr >  t
1	46.3333333	0.6047650	<.0001
2	48.3333333	0.6047650	<.0001
3	51.0000000	0.6047650	<.0001
4	44.3888889	0.7807483	<.0001

## Estimate - Must adjust F by hand

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	179.7060185	35.9412037	39.31	0.0002
Error	6	5.4861111	0.9143519		
Corrected Total	11	185.1921296			

R-Square	Coeff Var	Root MSE	y Mean
0.970376	2.012501	0.956217	47.51389

Source	DF	Type I SS	Mean Square	F Value	Pr > F
soap	3	71.9513889	23.9837963	26.23	0.0008
stain	2	107.7546296	53.8773148	58.92	0.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
soap	3	71.9513889	23.9837963	26.23	0.0008
stain	2	107.7546296	53.8773148	58.92	0.0001

LSD Test

	Mean	N	soap
A	51.0000	3	3
B	48.3333	3	2
C	46.3333	3	1
D	44.3889	3	4

$$F_0 = \frac{71.95/3}{5.49/5} = 21.84$$

$$Pvalue = 0.0027$$

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## Tukey's Test for Non-additivity

- Consider special type of interaction
- Assume following model (page 190-193)

$$y_{ij} = \mu + \tau_i + \beta_j + \gamma\tau_i\beta_j + \epsilon_{ij}$$

- $H_0 : \gamma = 0$
- Use regression approach to test significance
- Procedure
  - 1 Fit additive model  $y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$
  - 2 Obtain  $\hat{y}_{ij}$  and  $y_{ij} - \hat{y}_{ij}$
  - 3 Fit additive model  $\hat{y}_{ij}^2 = q_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$
  - 4 Regress  $y_{ij} - \hat{y}_{ij} = q_{ij} - \hat{q}_{ij}$

Partitioning  $SS_E$  into  $SS_N$  and remainder

The parameter  $\hat{\gamma}$  is the slope estimate

$$SS_{\text{non-additivity}} = \hat{\gamma}^2 \sum \sum (q_{ij} - \hat{q}_{ij})^2$$

$$F_0 = \frac{SS_N/1}{(SS_E - SS_N)/((a-1)(b-1) - 1)}$$

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## Example 5-2 from Montgomery

- Impurity in chemical product is affected by temperature and pressure. We will assume temperature is the blocking factor. The data is shown below. We will test for non-additivity.

Temp	Pressure				
	25	30	35	40	45
100	5	4	6	3	5
125	3	1	4	2	3
150	1	1	3	1	2

- Can use SAS to compute SS
- Must divide by proper degrees of freedom

$$F_0 = \frac{.0985/1}{1.9015/7} = .36$$

$F_0 < F_{1,7}$  - Do Not Reject.

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## SAS Procedures

```
options nocenter ls=75;

data impurity;
input trt blk y @@;
cards;
1 1 5 1 2 3 1 3 1 2 1 4 2 2 1 2 3 1 3 1 6 3 2 4 3 3 3
4 1 3 4 2 2 4 3 1 5 1 5 5 2 3 5 3 2
;

proc glm;
class blk trt;
model y=blk trt;
output out=resid1 r=res1 p=pred1;

data predsq;
set resid1;
predsq1 = pred1*pred1;

proc glm;
class blk trt;
model predsq1=blk trt;
output out=resid2 r=res2 p=pred2;

proc glm;
model res1=res2;
run;
```

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Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	34.93333333	5.82222222	23.29	0.0001
Error	8	2.00000000	0.25000000		
Corrected Total	14	36.93333333			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
blk	2	23.33333333	11.66666667	46.67	<.0001
trt	4	11.60000000	2.90000000	11.60	0.0021

Dependent Variable: res1

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.09852217	0.09852217	0.67	0.4266
Error	13	1.90147783	0.14626753		
Corrected Total	14	2.00000000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
res2	1	0.09852217	0.09852217	0.67	0.4266

Parameter	Estimate	Standard Error	t Value	Pr >  t
Intercept	-.000000000	0.09874800	-0.00	1.0000
res2	0.0369458128	0.04501655	0.82	0.4266

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## Random Block/Treatment Effects

- Could randomly select trts and/or blocks
- Do not need to worry about interaction
- Interaction considered random effect
- Interaction variance appears in all EMS
- Perform usual F-test (ratio of MS)
- Use Proc Mixed instead of Proc Glim
- Otherwise underestimate variability in trt means

## Choice of Sample Size

- Same as determining the number of blocks (*b*)
- Use same tables/procedures with *b* replacing *n*

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```
options nocenter ls=75;
```

```
data wash;
input stain soap y @@;
cards;
1 1 45 1 2 47 1 3 48 1 4 42
2 1 43 2 2 46 2 3 50 2 4 37
3 1 51 3 2 52 3 3 55 3 4 49
;
proc glm;
class stain soap;
model y = soap stain soap*stain;
random stain soap*stain;
run;
```

General Linear Models Procedure

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	264.9166667	24.08333333	.	.
Error	0	.	.	.	.
Corrected Total	11	264.9166667			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
SOAP	3	110.9166667	36.97222222	.	.
STAIN	2	135.1666667	67.58333333	.	.
STAIN*SOAP	6	18.83333333	3.13888889	.	.

Source Type III Expected Mean Square

SOAP Var(Error) + Var(STAIN\*SOAP) + Q(SOAP)  
STAIN Var(Error) + Var(STAIN\*SOAP) + 4 Var(STAIN)  
STAIN\*SOAP Var(Error) + Var(STAIN\*SOAP)

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```
options nocenter ls=75;

data wash;
input stain soap y @@;
cards;
1 1 45 1 2 47 1 3 48 1 4 42
2 1 43 2 2 46 2 3 50 2 4 37
3 1 51 3 2 52 3 3 55 3 4 49
;

proc glm;
class stain soap;
model y = soap stain;
random stain;
lsmeans soap / stderr tdiff;

proc mixed;
class stain soap;
model y = soap;
random stain;
lsmeans soap / stderr tdiff;
run;
```

11-26

The GLM Procedure

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	246.0833333	49.21666667	15.68	0.0022
Error	6	18.83333333	3.13888889		
Corrected Total	11	264.9166667			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
soap	3	110.9166667	36.97222222	11.78	0.0063
stain	2	135.1666667	67.58333333	21.53	0.0018

soap	y LSMEAN	Standard Error	Pr >  t	LSMEAN Number
1	46.33333333	1.0228863	<.0001	1
2	48.33333333	1.0228863	<.0001	2
3	51.00000000	1.0228863	<.0001	3
4	42.66666667	1.0228863	<.0001	4

Least Squares Means for Effect soap  
t for H0: LSMean(i)=LSMean(j) / Pr > |t|

		Dependent Variable: y			
i/j		1	2	3	4
1			-1.38257	-3.226	2.534715
			0.2161	0.0180	0.0444
2	1.382572			-1.84343	3.917286
	0.2161			0.1148	0.0078
3	3.226001	1.843429			5.760715
	0.0180	0.1148			0.0012
4	-2.53471	-3.91729	-5.76072		
	0.0444	0.0078	0.0012		

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The Mixed Procedure

Covariance Parameter Estimates

Cov Parm	Estimate
stain	16.1111
Residual	3.1389

Type 3 Tests of Fixed Effects

Effect	Num	Den	F Value	Pr > F
soap	3	6	11.78	0.0063

Least Squares Means

Effect	soap	Estimate	Standard Error	DF	t Value	Pr >  t
soap	1	46.3333	2.5331	6	18.29	<.0001
soap	2	48.3333	2.5331	6	19.08	<.0001
soap	3	51.0000	2.5331	6	20.13	<.0001
soap	4	42.6667	2.5331	6	16.84	<.0001

Differences of Least Squares Means

Effect	soap	_soap	Estimate	Standard Error	DF	t Value	Pr >  t
soap	1	2	-2.0000	1.4466	6	-1.38	0.2161
soap	1	3	-4.6667	1.4466	6	-3.23	0.0180
soap	1	4	3.6667	1.4466	6	2.53	0.0444
soap	2	3	-2.6667	1.4466	6	-1.84	0.1148
soap	2	4	5.6667	1.4466	6	3.92	0.0078
soap	3	4	8.3333	1.4466	6	5.76	0.0012

## RCBD with Replication

- What if multiple trt observation per block?
  - $b$  blocks
  - $a$  treatments
  - $n$  observations/block

$$y_{ijl} = \mu + \tau_i + \beta_j + \epsilon_{ijl} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \\ l = 1, 2, \dots, n \end{cases}$$

- When would this occur?
  - Have large field with very gradual slope
  - Blocks expensive but observations cheap
- Increases  $df_E$  (or allows interaction)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	$F$
Blocks	$SS_{Block}$	$b - 1$	$MS_{Block}$	
Treatment	$SS_{Treatment}$	$a - 1$	$MS_{Treatment}$	$F_0$
Error	$SS_E$	$abn - b - a + 1$	$MS_E$	
Total	$SS_T$	$abn - 1$		

## RCBD with Replication

- Usual diagnostics checks
- Replace  $b$  by  $bn$  in multiple comparisons or power
- Allows for easier assessment of additivity
  - More error degrees of freedom
  - Interaction and error not confounded
  - Can separate error and interaction SS

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	$F$
Blocks	$SS_{Block}$	$b - 1$	$MS_{Block}$	
Treatment	$SS_{Treatment}$	$a - 1$	$MS_{Treatment}$	$F_0$
Blk*Trt	$SS_{Blk*Trt}$	$(b - 1)(a - 1)$	$MS_{Blk*Trt}$	
Error	$SS_E$	$ab(n - 1)$	$MS_E$	
Total	$SS_T$	$abn - 1$		

## Example

You have been asked to design an experiment to compare four varieties of seed corn. You have a field consisting of sixteen subplots (in a 4x4 grid) at your disposal. If you were told that one side of the field is next to a highway and the side directly across from this one is next to a river, how would you design the experiment?

If we feel pretty certain that plots near the road or river will "behave" differently than plots in the middle, we might want to create  $b = 3$  blocks. Block 1 consists of the four plots along the road. Block 2 consists of the 4 plots along the river and Block 3 consists of the eight plots in the middle. Thus, we have two blocks which only have  $n = 1$  observation per treatment and one block that has  $n = 2$  observations per treatment.

- Statistical model is (with interaction)

$$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, 3, 4 \\ j = 1, 2, 3 \\ k = 1, \dots, n_j \end{cases}$$

where  $n_j = \begin{cases} 1 & \text{if } j = 1, 2 \\ 2 & \text{if } j = 3 \end{cases}$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	$F_0$
Blocks	$SS_{Block}$	2	$MS_{Block}$	
Interaction	$SS_{Trt*Blk}$	6	$MS_{Trt*Blk}$	
Treatment	$SS_{Treatment}$	3	$MS_{Treatment}$	$F_0$
Error	$SS_E$	4	$MS_E$	
Total	$SS_T$	15		

- If four blocks, cannot separate error and interaction
- $SS_E$  based on obs within block 3 (has replicates)